ON STRICT-DOUBLE-BOUND NUMBERS OF GRAPHS AND SUM OPERATIONS

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Abstract

For a poset $P = (X, \leq_P)$, the strict-double-bound graph of $P$ is the graph $sDB(P)$ on $V(sDB(P)) = X$ for which vertices $u$ and $v$ of $sDB(P)$ are adjacent if and only if $u \neq v$ and there exist elements $x, y \in X$ distinct from $u$ and $v$ such that $x \leq_P u \leq_P y$ and $x \leq_P v \leq_P y$. The strict-double-bound number $\zeta(G)$ of a graph $G$ is defined as $\min\{n; sDB(P) \cong G \cup K_n \text{ for some poset } P\}$. We consider strict-double-bound numbers in terms of sum operations. We obtain that for a connected graph $G$ with at least two vertices and a poset $P$ such that $sDB(P) \cong G \cup K_{\zeta(G)}$,

$$\zeta(G + mK_n) \leq \zeta(G) + (m - 1) \times \min\{|\text{Min}(P)|, |\text{Max}(P)|\}.$$  

We also obtain that $\zeta(G) \leq |S| + 2$ for a split graph $G$ with $V(G) = V(K_n) \cup S$.

Keywords and phrases: strict-double-bound graph, strict-double-bound number, sum operation.